

Boundary-layer control by electric fields: A feasibility study

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Abstract

A problem of great concern in aviation and submarine propulsion is the control of the boundary layer and, in particular, the methods to extend the laminar region as a means to decrease noise and fuel consumption. In this paper we study the flow of air along an airfoil when a layer of ionized gas and a longitudinal electric field are created in the boundary layer region. By deriving scaling solutions and more accurate numerical solutions we discuss the possibility of achieving significant boundary layer control for realistic physical parameters. Practical design formulas and criteria are obtained. We also discuss the perspectives for active control of the laminar-to-turbulent transition fluctuations by electromagnetic field modulation.

1 Boundary layers and boundary layer control

Whether a flow is laminar or turbulent, the effects of the viscosity of the fluid are greatest in regions close to solid boundaries. The region close to

the boundary, in which the velocity varies from zero, relative to the surface, up to its full value, is called the *boundary layer*. The concept of boundary layer introduced by Prandtl in 1904, was a most significant advance in fluid dynamics, in the sense that it simplified the study by separating the flow in two parts: (1) the region where velocity gradients are large enough to produce appreciable viscous forces - the boundary layer itself - and (2) the external region where viscous forces are negligible compared to other forces. From the computational point of view the concept of boundary layer also plays a significant role because, rather than having to deal with time-consuming general purpose finite-element codes, results of comparable precision may be obtained by fast and relatively simple finite-difference implicit algorithms.

When a fluid flows past a solid body, an airfoil for example, a *laminar boundary layer* develops, in general only for a very small distance near the leading edge, followed by a transition to a *turbulent boundary layer*. Nevertheless, because near the solid wall velocity fluctuations must die out, below the turbulent region there always is a *laminar sub-layer* which in general is very small (of the order of a micrometer). The transition from the laminar to the turbulent region is controlled by the *local Reynolds number*, defined in terms of the effective thickness of the boundary layer. It also depends on the smoothness of the surface and on the external perturbations. The *skin friction drag* is proportional to the gradient of the longitudinal velocity at the solid boundary. Because of the mixing properties of the turbulent layer, the gradient in the laminar sub-layer is much greater than the gradient at a fully laminar layer. Therefore transition from a laminar to a turbulent layer greatly increases the skin friction drag. Another effect to be taken into account is the *separation* of the boundary layer, which occurs at points where the pressure gradient along the surface reverses sign. The eddies, generated by the resulting reverse flow, disturb the flow and form a *wake* where energy dissipation decreases the pressure, thereby increasing the *pressure drag*.

Because of the very large ratio between laminar and turbulent skin friction drag, much effort has been devoted to develop techniques to delay the transition as a means of decreasing fuel consumption and noise. Care should however be taken because, in general, a turbulent boundary layer is more stable towards separation than a laminar boundary layer. Some of the active control techniques that have been proposed include suction of slow-moving fluid through slots or a porous surface, use of compliant walls and wall cooling (or wall heating for liquids). Injection of fast-moving fluid, on the other

hand, is effective in avoiding separation but it increases turbulence. Most of these ideas are fairly old (see for example [1],[2]) however, in view of their interest for the applications and to obtain a more accurate characterization of the physical mechanisms, studies of boundary layer control using these aerodynamic methods are still, at present, being vigorously pursued (see for example [3] [4] [5] [6] and papers in [7])

Another class of techniques for active boundary layer control consists in acting on the flow by means of electromagnetic forces. Here different techniques should be envisaged according to whether the fluid is weakly conducting (an electrolyte like seawater or an ionized gas) or a good conductor (like a liquid metal). Proposals for boundary layer control by electromagnetic forces are also relatively old and trace its origin at least to the papers of Gailitis and Lielausis[8], Tsinober and Shtern[9] and Moffat[10] in the sixties. Interest in these techniques has revived in recent years and some more accurate calculations and experimental verifications have been carried out, mostly in the context of electrolyte fluids[11] [12].

In this paper we will be concerned with the flow of air along an airfoil when a layer of ionized gas is created on the boundary layer region. Local ionization of the air along the airfoil is not practical from the technological point of view, therefore we will assume that a stream of ionized air (or some other ionized gas) is injected through a backwards facing slot placed slightly behind the stagnation point (Fig.1). The body force that we consider to be acting in the ionized fluid is a longitudinal (along the flow) electric field created by a series of plate electrodes transversal to the flow and placed inside the airfoil with the edges on the airfoil surface.

The emphasis of our study is on finding physically reasonable ranges of parameters and analytic approximations that might lead to simple designing procedures. For this purpose, before the numerical calculation of Section 3, we dedicate some time to the study of scaling solutions and analytical approximations.

The provisional conclusions of our study are that it is possible to use this technique to control the profile of the boundary layer laminar region. With the rates of ionization that are needed and the injection method, it is probably unrealistic to expect that the laminar region may be extended over all the airfoil in normal (aviation) working conditions. Therefore this method should be used in conjunction with methods for control of turbulent boundary layers (riblets, large-eddy breakups, additives, etc.) in the rear part

of the airfoil. Also the injection of the stream of ionized gas in the leading edge may, by increasing the velocity component normal to the airfoil, create turbulence. Therefore it seems advisable to have a compensating suction region after the injection slot. The ionization rate will also improve if the gas extracted through the suction region is recycled through the ionizer.

Notice also that, once the fluid in the boundary layer is ionized, large scale velocity fluctuations may be detected by a few local probes. This raises the possibility of obtaining a negative feedback effect by an appropriate time-dependent modulation of the electric field. By controlling the growth of the velocity fluctuation in the transition region, a further extension of the laminar region may be obtained. This is briefly discussed in the last section of the paper.

The overall conclusion is that, when used in conjunction with other techniques, as explained above, the method of boundary layer control by electric fields might be interesting from the fuel consumption point of view. This study was carried out as a preparation for an experiment being set up in our Mechatronics Laboratory.

2 Ionized boundary layers with electric fields

2.1 The boundary layer equations

We use orthogonal curvilinear coordinates with \tilde{x} parallel to the surface along the flow and \tilde{y} normal to the surface. If $\kappa\delta$ is small (κ denoting the curvature and δ the boundary layer thickness) the conservation and momentum equations in the incompressible fluid approximation may be written

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0 \quad (1)$$

$$\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{u}}{\partial \tilde{y}} = -\frac{1}{\tilde{\rho}_m} \frac{\partial \tilde{p}}{\partial \tilde{x}} + \tilde{\nu} \left(\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} \right) + \frac{1}{\tilde{\rho}_m} \tilde{\sigma}_e(\tilde{x}, \tilde{y}) \tilde{E}_x(\tilde{x}, \tilde{y}) \quad (2)$$

$$\frac{\partial \tilde{v}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{v}}{\partial \tilde{y}} = -\frac{1}{\tilde{\rho}_m} \frac{\partial \tilde{p}}{\partial \tilde{y}} + \tilde{\nu} \left(\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2} \right) + \frac{1}{\tilde{\rho}_m} \tilde{\sigma}_e(\tilde{x}, \tilde{y}) \tilde{E}_y(\tilde{x}, \tilde{y}) \quad (3)$$

\tilde{u} and \tilde{v} are the components of the fluid velocity field along the \tilde{x} and \tilde{y} directions. $\tilde{\rho}_m$ is the mass density, $\tilde{\sigma}_e$ the electric charge density and \tilde{E} an

applied electric field. The tilde denotes quantities in physical dimensions to be distinguished from the adimensional quantities defined below. We consider typical values $L_r, \delta_r, U_r, \rho_r, \nu_r, \sigma_r, E_r$ as reference values for, respectively, the airfoil width, the boundary layer thickness, the fluid velocity, the fluid mass density, the kinematic viscosity, the fluid charge density and the electric field. Then we define the adimensional quantities

$$t = \tilde{t} \frac{U_r}{L_r}, \quad x = \frac{\tilde{x}}{L_r}, \quad y = \frac{\tilde{y}}{\delta_r}, \quad u = \frac{\tilde{u}}{U_r}, \quad v = \frac{\tilde{v} L_r}{U_r \delta_r} \quad (4)$$

$$\rho_m = \frac{\tilde{\rho}_m}{\rho_r}, \quad p = \frac{\tilde{p}}{\rho_r U_r^2}, \quad R_L = \frac{U_r L_r}{\nu_r} \quad (5)$$

$$\nu = \frac{\tilde{\nu}}{\nu_r}, \quad \sigma = \frac{\tilde{\sigma}}{\sigma_r}, \quad E = \frac{\tilde{E}}{E_r} \quad (6)$$

In general $R_L \gg 1$. Neglecting terms of order $\frac{1}{R_L}$ and $\frac{\delta_r^2}{L_r^2}$ we obtain, for stationary solutions ($\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0$)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (7)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_m} \frac{\partial p}{\partial x} + \nu \omega \frac{\partial^2 u}{\partial y^2} + \gamma \frac{1}{\rho_m} \sigma(x, y) E_x(x, y) \quad (8)$$

$$\frac{\partial p}{\partial y} = \frac{\delta_r}{L_r} \gamma \sigma(x, y) E_y(x, y) \quad (9)$$

where $\omega = \frac{L_r^2}{\delta_r^2 R_L} = \frac{L_r \nu_r}{\delta_r^2 U_r}$ and $\gamma = \frac{L_r \sigma_r E_r}{U_r^2 \rho_r}$. Unless the electric field component normal to the airfoil is very large, one has $\frac{\partial p}{\partial y} \approx 0$ and the pressure term in the second equation may be expressed in terms of the fluid velocity u_e far away from the airfoil

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{\partial u_e}{\partial x} + \nu \omega \frac{\partial^2 u}{\partial y^2} + \gamma \frac{1}{\rho_m} \sigma(x, y) E_x(x, y) \quad (10)$$

To take into account turbulence effects one should also replace in (10) the velocity fields u and v by $u + u'$ and $v + v'$, u' and v' being fluctuation fields with zero mean, $\overline{u'} = 0$, $\overline{v'} = 0$. The effect of the turbulent field on the mean flow is now obtained by taking mean values. In a two-dimensional turbulent boundary layer the dominant eddy stress is $\overline{-u'v'}$. Assuming the eddy shear stress $\overline{-u'v'}$ and the mean rate of strain $\frac{\partial u}{\partial y}$ to be linearly related

$$\overline{-u'v'} = \epsilon \frac{\partial u}{\partial y} \quad (11)$$

one obtains finally

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e \frac{\partial u_e}{\partial x} + \nu \omega \frac{\partial}{\partial y} \left(\beta \frac{\partial u}{\partial y} \right) + \gamma \frac{1}{\rho_m} \sigma(x, y) E_x(x, y) \quad (12)$$

with

$$\beta = 1 + \frac{\epsilon}{\omega} \quad (13)$$

being, in general, a function of y through the dependence of the eddy viscosity ϵ on the local velocity field. β should be obtained from a turbulence model.

To analyze the scaling solutions and for the numerical calculations in Sect. 3 we define a stream function ψ and make the following change of variables

$$\eta = \left(\frac{u_e}{\nu \omega} \right)^{\frac{1}{2}} \frac{y}{\xi(x)} \quad (14)$$

$$\psi = (u_e \nu \omega)^{\frac{1}{2}} \xi(x) f(x, \eta) \quad (15)$$

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (16)$$

The continuity equation (7) is automatically satisfied by (16) and one is left with

$$\frac{\partial}{\partial \eta} \left(\beta \frac{\partial^2 f}{\partial \eta^2} \right) + \xi \frac{\partial \xi}{\partial x} f \frac{\partial^2 f}{\partial \eta^2} + \frac{\xi^2}{u_e} \frac{\partial u_e}{\partial x} + \xi^2 \left(\frac{\partial^2 f}{\partial \eta^2} \frac{\partial f}{\partial x} - \frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial \eta \partial x} \right) = -\frac{\gamma}{u_e^2 \rho_m} \xi^2(x) \sigma(x, \eta) E_x(x, \eta) \quad (17)$$

2.2 Scaling solutions

We assume that the electric field to be created by a series of plate electrodes along the z-direction, that is transversal to the fluid flow. For this electrode geometry the mean electric field in the x-direction may be parametrized by

$$\overline{E_x} = g(x) \frac{l(x)}{l^2(x) + y^2} \quad (18)$$

where x and y are the adimensional coordinates defined in (4). $E_0 = \frac{g(x)}{l(x)}$ is the field at $y = 0$, controlled by the potential differences between the electrodes, and $l(x)$ is of the order of the electrode spacing. For thin (laminar) boundary layers the field E_x may with good approximation be considered to be independent of y throughout the boundary layer thickness, as long as the appropriate charge density profile is chosen (see below).

For the main application we are addressing, ionized air would be injected through a slot near the leading edge of the airfoil, being then carried along the airfoil surface by the flow. The steady-state charge distribution in the boundary layer is obtained from the continuity equation

$$\frac{\partial \sigma}{\partial x} u + \frac{\partial \sigma}{\partial y} v = j \quad (19)$$

j being the source of electric charge. For a point source at the position (x_0, y_0) , that is $j = c(x_0, y_0) \delta(x - x_0) \delta(y - y_0)$, the solution is

$$\sigma(x, y) = c(x_0, y_0) \delta(\psi(x, y) - \psi(x_0, y_0)) \theta(x - x_0) \quad (20)$$

ψ being the stream function defined before. Then, for a column of ionized air injected at a backwards facing angle through a slot placed at x_0 , behind the stagnation point, each point acts as a point source of intensity proportional to the local fluid velocity. Furthermore the intensity of the effective source

is depleted up the column. Taking the depletion effect into account, one obtains by integration of Eq.(20)

$$\sigma(x, y) = \sigma_0 (1 - d_1 \psi(x, y)) \theta(1 - d_1 \psi(x, y)) \theta(x - x_0) \quad (21)$$

σ_0 is the injection intensity and d_1 characterizes the rate of depletion. The conclusion is that the charge density is maximum at the airfoil surface, decreasing to zero at a distance that depends on the fluid dynamics and the injection regime. In numerical simulations one may easily use the fairly accurate equation (21) for the charge density profile. Here however, the dynamically-dependent charge density profile will be parametrized by the simpler formula

$$\sigma(x, y) = \sigma_0 \left(1 - \frac{u}{u_e}\right) \quad (22)$$

We now look for scaling solutions of (17). A scaling solution is one for which f is only a function of η . Eq.(17) becomes

$$(\beta f'')' + \xi \dot{\xi} f f'' + \frac{\xi^2}{u_e} \frac{\partial u_e}{\partial x} = -\frac{\gamma}{u_e \rho_m} \xi^2(x) \sigma_0 (1 - f') g(x) \frac{l(x)}{u_e l^2(x) + \omega \nu \xi^2(x) \eta^2} \quad (23)$$

with boundary conditions

$$f(0) = f'(0) = 0 \quad f'(\infty) = 1 \quad (24)$$

where, for simplicity, we have denoted $f' \equiv \frac{\partial f}{\partial \eta}$ and $\dot{\xi} = \frac{\partial \xi}{\partial x}$.

Let the pressure be approximately constant for length scales L of the order of the airfoil, that is $\frac{\partial u_e}{\partial x} \approx 0$. Let also β be a constant. This is the case for the laminar part of the boundary layer. Then the factorized nature of Eq.(23) implies that solutions exist only if

$$\xi^{-2}(x) = \frac{2}{c_1} \frac{\dot{\xi}(x)}{\xi(x)} = \frac{1}{c_3} \frac{g(x)}{\xi(x)} = c_4 l^{-2}(x) \quad (25)$$

c_1 , c_3 and c_4 being constants. Therefore

$$\begin{aligned} \xi(x) &= \sqrt{c_1 x + c_2} \\ g(x) &= \frac{c_3}{\xi(x)} \\ l(x) &= \sqrt{c_4 \xi(x)} \end{aligned} \quad (26)$$

There are two physically interesting situations. The one with $c_1 \neq 0$ $c_2 = 0$ and the one with $c_1 = 0$ $c_2 \neq 0$. The first one corresponds to a boundary layer starting at $x = 0$ and growing with $x^{\frac{1}{2}}$ and the second to a constant thickness boundary layer. The first one corresponds to an equation

$$f'''(\eta) + \frac{1}{2}f(\eta)f''(\eta) + (1 - f'(\eta)) \frac{\varphi_1}{\varphi_2^2 + \eta^2} = 0 \quad (27)$$

with $c_1 = \beta$, $c_2 = 0$, $\varphi_1 = \frac{\gamma\sigma_0 c_3 \sqrt{c_4}}{u_e \beta \rho_m \nu \omega}$, $\varphi_2 = \sqrt{\frac{u_e c_4}{\nu \omega}}$ and the second to

$$f'''(\eta) + (1 - f'(\eta)) \frac{a}{b^2 + \eta^2} = 0 \quad (28)$$

with $c_1 = 0$, $c_2 \neq 0$, $a = \frac{\gamma\sigma_0 c_3 \sqrt{c_4}}{u_e \beta \rho_m \nu \omega}$, $b = \sqrt{\frac{u_e c_4}{\nu \omega}}$.

In the first case one chooses $c_2 = 0$ to obtain a boundary layer starting at $x = 0$. The scaling hypothesis requires then an electric field that is singular at $x = 0$, $y = 0$ ($E_x \sim x^{-1}$). In any case this electric field solution is not very interesting for our purposes because it leads to a boundary layer growth of $x^{\frac{1}{2}}$, as in the free force Blasius solution. Therefore it will be more interesting to consider a small field free region in the leading edge of the airfoil and match the Blasius solution there with the constant thickness solution of Eq.(28).

Gailitis and Lielausis[8] have also obtained a theoretical solution of constant thickness. However they consider a different force field distribution and no dependence of the fluid charge density on the boundary layer dynamics. Therefore their boundary layer profile has a very different behavior.

The solution of Eq.(28) is easily obtained by numerical integration (see below). Notice however that with the replacement

$$\phi(\eta) = 1 - f'(\eta) \quad (29)$$

and choosing $c_4 = \frac{\nu \omega}{u_e}$, which is a simple rescaling of ξ , Eq.(28) becomes the zero-eigenvalue problem for a Schrödinger equation in the potential $a/(1+\eta^2)$,

$$-\phi''(\eta) + \frac{a}{1 + \eta^2} \phi(\eta) = 0 \quad (30)$$

One may use the well-known WKB approximation to obtain

$$f'(\eta) = 1 - \frac{(1 + \eta^2)^{\frac{1}{4}}}{(\eta + \sqrt{1 + \eta^2})^{\sqrt{a}}} \quad (31)$$

Eq.(31) is a very good approximation to the exact solution for $a \geq 1$ (see Fig. 2). Fig.3 shows the effective boundary layer thickness as a function of a . The effective boundary layer thickness δ^* is defined here as the value of η at which the velocity u reaches 0.95 of its asymptotic value u_e . A very fast thinning of the boundary layer is obtained (several orders of magnitude) for a relatively short range of the a parameter. Fig.3 shows the variation of δ^* for small a . For large a (and small δ^*) one has the asymptotic formula

$$\delta^* \simeq \frac{2.9957}{\sqrt{a}}$$

which is obtained from Eq.(31).

If the longitudinal electric field E_x is assumed to be a constant (E_0) throughout the boundary layer thickness, with the same charge profile, the solution is even simpler, namely

$$f'(\eta) = 1 - e^{-\eta\sqrt{h}} \quad (32)$$

with $\xi = \sqrt{c_2}$ and

$$h = \frac{\gamma c_2 \sigma_0 E_0}{\beta u_e^2 \rho_m} \quad (33)$$

Again, since ξ is a constant, this is not fully realistic because it leads to a constant thickness boundary layer.

For reference values of the physical quantities in Eqs.(4-6) we take

$$\begin{aligned} U_r &= 100 \text{ m s}^{-1} \\ L_r &= 1 \text{ m} \\ \delta_r &= 10^{-3} \text{ m} \\ \rho_r &= 1.2 \text{ Kg m}^{-3} \\ E_r &= 500 \text{ V cm}^{-1} \\ \sigma_r &= 15 \text{ } \mu\text{C cm}^{-3} \\ \nu_r &= 1.5 \times 10^{-5} \text{ m}^2 \text{ } \textit{textrnormals}^{-1} \end{aligned} \quad (34)$$

For these reference values, the adimensional constants ω and γ defined after Eq.(9) are

$$\begin{aligned} \omega &= 0.15 \\ \gamma &= 62.499 \end{aligned} \quad (35)$$

For comparison we mention that in the classical force-free Blasius solution, and for these reference parameters, the y -coordinate y^* corresponding to δ^* (that is, the point at which $\frac{u}{u_e} = 0.95$) is

$$y^* = 1.55 \times 10^{-3} \sqrt{x} \quad (36)$$

Stability of a laminar boundary layer cannot safely be guaranteed for local Reynold numbers greater than about 10^3 . Therefore requiring

$$R_S = \frac{\tilde{u}_e \tilde{y}^*}{\tilde{\nu}} \simeq 10^3 \quad (37)$$

one obtains, for the reference parameters, $\tilde{y}^* \simeq 0.15$ mm. Using (34) the conclusion is that, for these parameters, the laminar part of a force-free boundary layer is only of the order of 1 cm, just a tiny portion of a typical wing.

Now we use the scaling solutions (32) and (31) to obtain an estimate of the effects of a longitudinal electric field. For the constant field case (32) from

$$f'(\delta^*) = 1 - e^{-\delta^* \sqrt{h}} = 0.95$$

and

$$y^* = \delta^* \sqrt{c_2} \sqrt{\frac{\nu \omega}{u_e}} = 0.15$$

using (33) one obtains

$$\sigma_0 = 0.957$$

That is, to insure a constant thickness boundary layer with local Reynolds number $R_S = 10^3$ (at the point where $\frac{u}{u_e} = 0.95$), one needs a charge density $\tilde{\sigma}_0$ at $y = 0$, in physical units (and for the reference values of the kinematical parameters)

$$\tilde{\sigma}_0 = \sigma_0 \sigma_r = 14.36 \mu\text{C cm}^{-3}$$

For the variable field case (31) the estimate depends on the separation of the electrodes. Taking $l(x) = 10$, that is an electrode separation of the order of one centimeter, and the references values for all quantities except for the charge density (namely $E_0 = \frac{g(x)}{l(x)} = 1$, $u_e = 1$, etc.) one obtains $c_4 = 0.15$, $g(x) = 10$, $c_3 = g(x)\xi(x) = 258.2$, $\sqrt{c_2} = \xi(x) = 25.8$, and requiring

$$\begin{aligned} y^* &= \delta^* \sqrt{c_2} \sqrt{\frac{\nu \omega}{u_e}} = 0.15 \\ f'(\delta^*) &= 0.95 \end{aligned}$$

one finally obtains $a = 39887.77$ leading to

$$\sigma_0 = 0.957$$

the same estimate as above. The large value of a that is obtained shows that the WKB expression (31) is a good approximation for physically interesting parameter values. On the other hand the fact that the same charge density estimate is obtained both in the constant-field and the variable-field cases, shows that it is realistic to consider the field as approximately constant throughout the laminar boundary layer thickness, as long as a variable charge profile (21) or (22) is used.

The above estimates were obtained using the reference values for the kinematic variables. For other values we have the following designing formula (in normalized units)

$$\sigma_0 E_0 = 0.957 \frac{u_e^3 \rho_m}{10^{-6} R_S^2 \nu} \quad (38)$$

3 Numerical results

For the numerical solution of Eq.(17), with σ given by Eq.(22), we use an implicit finite-difference technique ([14] - [16]). Define $F(x, \eta)$ by

$$F(x, \eta) = \frac{\partial f}{\partial \eta} \quad (39)$$

and

$$\begin{aligned} a_1 &= \frac{1}{\beta} \left(\frac{\partial \beta}{\partial \eta} + \xi \frac{\partial \xi}{\partial x} f + \xi^2 \frac{\partial f}{\partial x} \right) \\ a_2 &= -\frac{\gamma}{\beta u_e^2 \rho_m} \xi^2 E_x \sigma_0 \\ a_3 &= -\frac{\xi^2}{\beta} F \\ a_4 &= \frac{\gamma}{u_e^2 \rho_m \beta} \xi^2 E_x \sigma_0 + \frac{\xi^2}{u_e \beta} \frac{\partial u_e}{\partial x} \end{aligned} \quad (40)$$

Then Eq.(17) becomes

$$\frac{\partial^2 F}{\partial \eta^2} + a_1 \frac{\partial F}{\partial \eta} + a_2 F + a_3 \frac{\partial F}{\partial x} + a_4 = 0 \quad (41)$$

The derivatives are replaced by finite-difference quotients with a variable grid spacing concentrated near $\eta = 0$, where F changes more rapidly. Let $k > 1$

be the ratio between two successive grid spacings in the η -direction.

$$k = \frac{\eta_{i+1} - \eta_i}{\eta_i - \eta_{i-1}}$$

Then

$$\begin{aligned} \left(\frac{\partial^2 F}{\partial \eta^2}\right)_{i+1,j} &= 2 \frac{F_{i+1,j+1} + k F_{i+1,j-1} - (1+k) F_{i+1,j}}{\Delta_2} \\ \left(\frac{\partial F}{\partial \eta}\right)_{i+1,j} &= \frac{F_{i+1,j+1} - k^2 F_{i+1,j-1} - (1-k^2) F_{i+1,j}}{\Delta_1} \\ \left(\frac{\partial F}{\partial x}\right)_{i+1,j} &= \frac{F_{i+1,j} - F_{i,j}}{\Delta x} \\ \Delta_1 &= \eta_{j+1} - \eta_j + k^2(\eta_j - \eta_{j-1}) \\ \Delta_2 &= (\eta_{j+1} - \eta_j)^2 + k(\eta_j - \eta_{j-1})^2 \end{aligned}$$

Substitution in Eq.(41) yields

$$A_j F_{i+1,j+1} + B_j F_{i+1,j} + D_j F_{i+1,j-1} + G_j = 0 \quad (42)$$

with

$$\begin{aligned} A_j &= \frac{2}{\Delta_2} + \frac{a_1}{\Delta_1} \\ B_j &= \frac{-2(1+k)}{\Delta_2} - \frac{a_1(1-k^2)}{\Delta_1} + a_2 + \frac{a_3}{\Delta x} \\ D_j &= \frac{2k}{\Delta_2} - \frac{a_1 k^2}{\Delta_1} \\ G_j &= a_4 - a_3 \frac{F_{i,j}}{\Delta x} \end{aligned}$$

The boundary conditions at $\eta = 0$ and $\eta \rightarrow \infty$ are known

$$\begin{aligned} f(i, 1) &= F(i, 1) = 0 \\ F(i, N) &= 1 \end{aligned}$$

where N is the largest label of the grid, in the η -coordinate, chosen to be sufficiently large.

Because of the tridiagonal nature of (42) the solution in the line $i + 1$ is obtained by the two-sweep method, the recursion relations being

$$\begin{aligned} F_{i+1,j} &= \alpha_j F_{i+1,j+1} + \beta_j \\ \alpha_j &= -\frac{A_j}{B_j + D_j \alpha_{j-1}} \\ \beta_j &= -\frac{G_j + D_j \beta_{j-1}}{B_j + D_j \alpha_{j-1}} \end{aligned}$$

with $\alpha_1 = 0$ and $\beta_1 = 0$.

To start the integration process there are basically two methods. In the first the integration is performed from left to right in the x -coordinate with

the grid extended to the left of the airfoil, where the flow is known. With the solution known in the line i , the coefficients A_j to G_j for Eq.(42) are computed at the point (i, j) . Notice that $f(i, j)$ is obtained by integration of the solution F .

$$f(i, \eta) = \int_0^\eta F(i, \zeta) d\zeta$$

The integration now proceeds along the lines, from left to right. After a complete pass the process is restarted using now for the calculation of the coefficients A_j , B_j , D_j and G_j the old values of F at $(i + 1, j)$. The process is repeated several times until the solution stabilizes.

In the second method, which is the one we actually use, the integration process starts from an approximate solution. The scaling solutions derived in Sect.2 are particularly useful for this purpose.

For our calculations we considered an electric field parametrized as in Eq.(18), namely

$$E_x = E_0 \frac{\frac{u_e l^2}{\nu \omega}}{\frac{u_e l^2}{\nu \omega} + \xi^2(x) \eta^2}$$

with $\frac{u_e l^2}{\nu \omega} = 666.66$ which corresponds to $l = 10$, $u_e = 1$ and $\nu \omega = 0.15$. Notice that for these parameters, as pointed out in Sect.2, the electric field has only a small variation throughout the boundary layer region. For the scaling function we take $\xi(x) = \sqrt{x}$ and consider $\beta = 1$. Then all results depend only on the variable S

$$S = \frac{1}{62.499} \frac{\gamma}{u_e^2 \rho_m} \sigma_0 E_0$$

($S = 1$ when all quantities take the reference values).

In Fig.4 we show a contour plot of the numerical solution for $f'(x, \eta)$ ($= \frac{u}{u_e}$) when $S = 0.6$. From the x -dependence of the numerical solutions we may compute the effect of the electric field in extending the laminar part of the boundary layer. By defining, as in Sect.2, the length of the laminar part as the x -coordinate corresponding to a local Reynolds number of 10^3 and denoting by x_0 ($\frac{u}{u_e} = 0.95$) the force-free value we have obtained for the ratio

$$R = \frac{x}{x_0}$$

the results shown in Fig.5. For $S = 0$ we obtain the Blasius solution and as we approach $S = 0.957$, corresponding to the scaling solution, the ratio

diverges. The matching of the results in the force-free and scaling limits is a good check of the numerical algorithm. A clear indication of the results in Fig.5 is that not much improvement is obtained unless one is able to obtain ionization charge densities of the order of the reference value σ_r .

4 Discussion and conclusions

In this paper we have concentrated on controlling the profile of the boundary layer. The profile has a direct effect on the laminar or turbulent nature of the flow which, in a simplified manner, we estimated by a local Reynolds number (37) defined as a function of the effective thickness. Another relevant aspect, of course, is the active control of the transition instabilities that can be achieved by electromagnetic body forces on the charged fluid.

Turbulence and transition to turbulence are three-dimensional phenomena. However, for the large scale small amplitude (Tollmien-Schlichting) fluctuations, that first appear in the transition region, a two-dimensional model is a reasonable approximation. In Eqs.(1-3) we make as before the change of variables (4-6), neglect terms of order $\frac{1}{R_L}$, $\frac{\delta_r^2}{L_r^2}$ and $\frac{\delta_r}{L_r}$ and split the velocity and electric fields into

$$\begin{aligned} u &= \bar{u} + u' \\ v &= \bar{v} + v' \\ E_x &= \bar{E}_x + E'_x \end{aligned}$$

where \bar{u} , \bar{v} , \bar{E}_x are the steady-state solutions and u' , v' , E'_x the time-dependent components. Because of the continuity equation

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} = 0$$

we may define a fluctuation stream function χ

$$u' = \frac{\partial \chi}{\partial y}, \quad v' = -\frac{\partial \chi}{\partial x}$$

Now we assume the fluctuation to be a (small-amplitude) wave-like function of x , y and t

$$\chi(x, y, t) = F(y)e^{i(\alpha x - \theta t)}$$

The imaginary parts of θ and α control, respectively, the growth rates of temporal and spatial fluctuations. The (modulation) electric field is assumed to have a similar form

$$E'_x = E e^{i(\alpha x - \theta t)}$$

One now obtains

$$\left(i\bar{u}\alpha + \frac{\partial \bar{u}}{\partial x} - i\theta \right) \frac{\partial F}{\partial y} - i\alpha \frac{\partial \bar{u}}{\partial y} F + \bar{v} \frac{\partial^2 F}{\partial y^2} = \nu \omega \frac{\partial^3 F}{\partial y^3} + \frac{\gamma}{\rho_m} \sigma E$$

The conclusion is that a space-time modulation of the electric field, with the appropriate phase, is equivalent to an effective viscous damping effect which delays the growth of the transition region instability. For this to be effective one needs to detect the phase of the wave instabilities by electromagnetic probes. Absolute synchronization of the feedback electric modulation is however not so critical as in acoustic noise cancelation, because here the objective is only to obtain an effective damping effect. The simplified treatment of the transition instabilities is justified by the fact that it is only for the small amplitude large scale fluctuations that one may hope to be able to detect the phase with some reasonable accuracy.

The kinematic reference parameters defined in (34) correspond to typical aviation conditions. The conclusion, both from the scaling solutions in Sect.2 and the numerical results in Sect.3, is that, to obtain a significant controlling effect on the boundary layer by this method, the charge density σ_0 (at $y = 0$) should be of the order of the reference charge density ($\sigma_r = 15 \mu\text{C cm}^{-3}$). This charge density corresponds to about 50 times the ion concentration a few centimeters away from the emitter of a commercial table-top negative corona discharge air purifier with a power of less than 6 watts. Therefore, it seems technically feasible to achieve a significant boundary layer control by this method. Another possibility would be to use, instead of air, some other easier to ionize gas. This could then be partially recovered and recycled by suction.

As explained in the introduction and because of the perturbation induced by the injection method, it seems advisable to use this method in conjunction with suction and passive control in the rear part of the airfoil. Even if a fully laminar boundary layer may never be completely achieved, just remember that any small improvement becomes, in the long run, quite significant in terms of fuel consumption.

The formula (38), derived from the scaling solutions, provides rough design estimates. Better control over design parameters we hope to obtain from the experimental work.

5 Figure captions

Fig.1 Airfoil transversal cut showing ionized air injection, suction pump and plate electrodes.

Fig.2 Exact (—) and approximate (···) constant thickness scaling solution $f'(\eta)$.

Fig.3 Effective boundary layer thickness δ^* ($f'(\delta^*) = \frac{u}{u_e} = 0.95$) for the constant thickness scaling solution.

Fig.4 Contour plot of $f'(x, \eta)$ for $S = 0.6$.

Fig.5 Ratio of boundary layer laminar regions with and without electric field control.

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Fig. 1

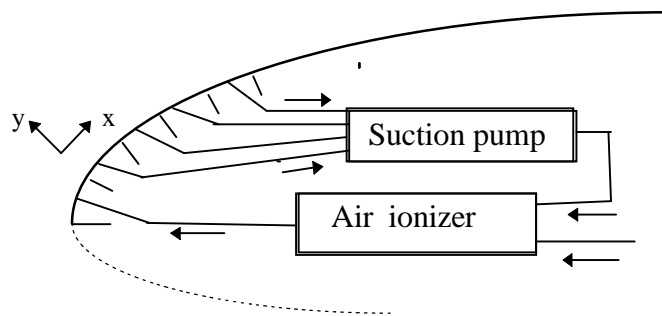


Fig.2

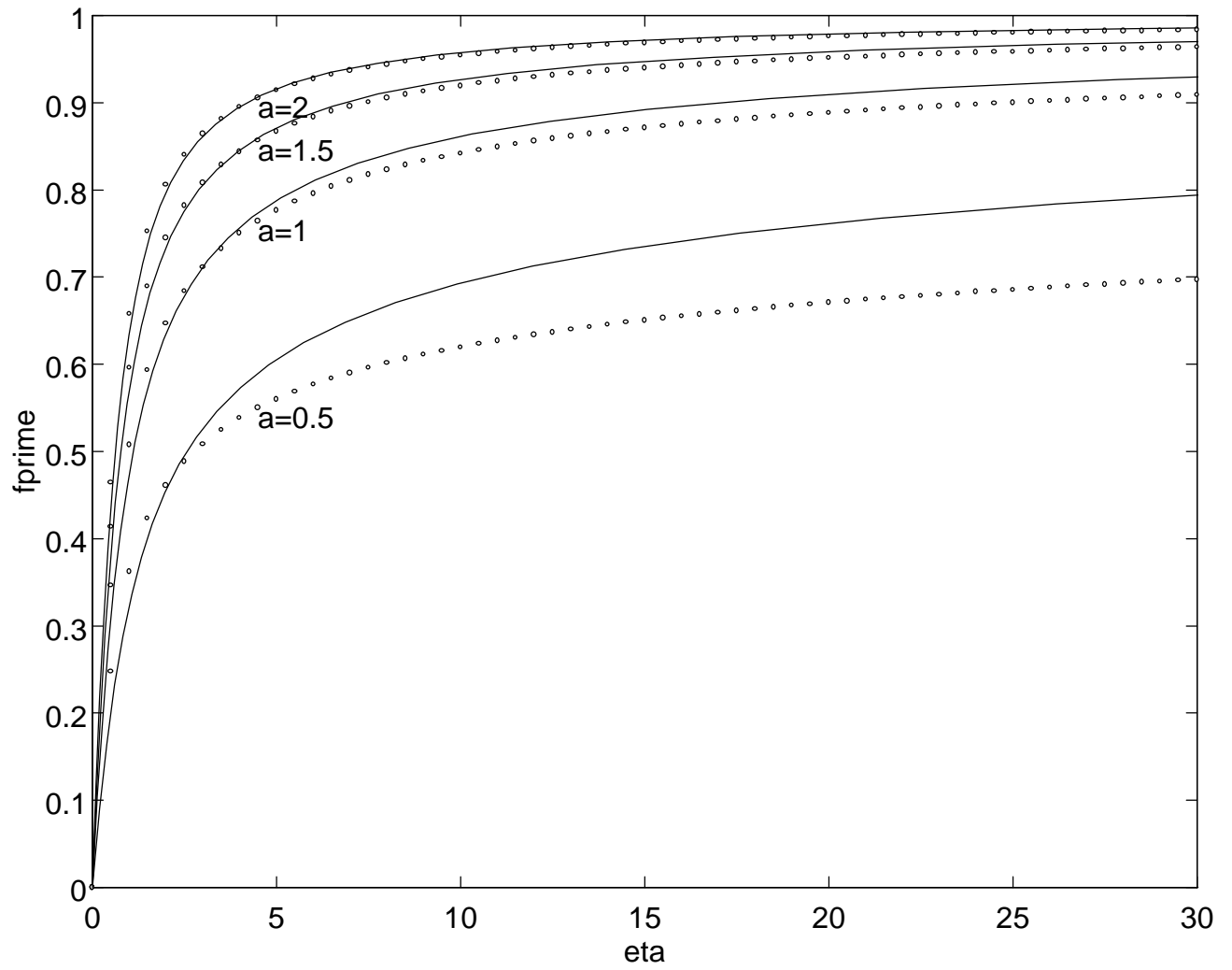


Fig.3

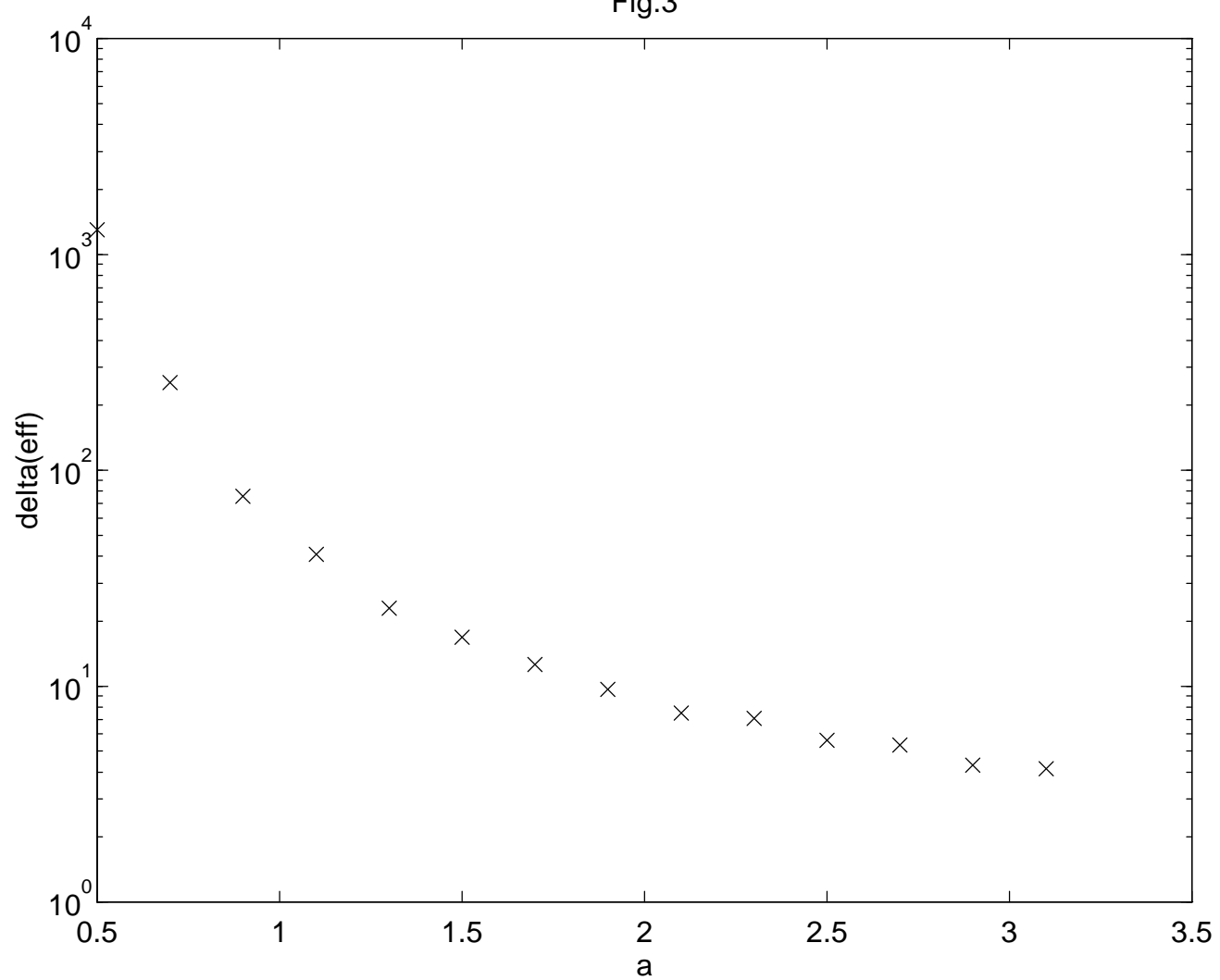


Fig.4

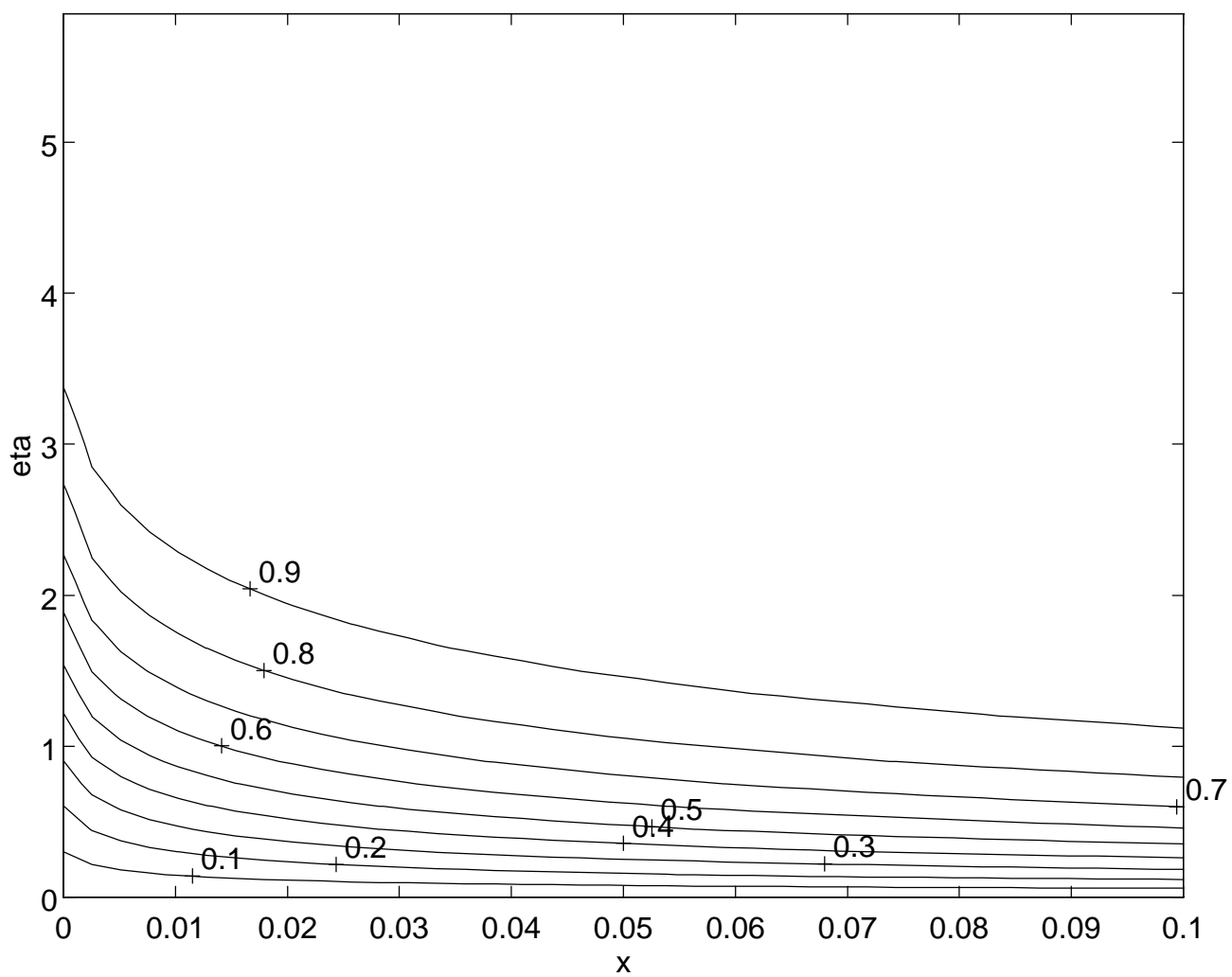


Fig.5

